Assignment 2

Coverage: 15.2 and 15.3 in Text.

Exercises: 15.2. no 20, 22, 27, 35, 39, 44, 48, 55, 61, 65, 79. 15.3. no 3, 6, 10, 11, 13, 15, 17, 21, 23, 26, 30.

Submit 15.2 no 27, 48, 65, 15.3 no 15, 21 by Sept. 27

Supplementary Problems

1. Let S be a non-empty set in $\mathbb{R}^n$. Define its characteristic function χ_S to be $\chi_S(\mathbf{x}) = 1$ for $\mathbf{x} \in S$ and $\chi_S(\mathbf{x}) = 0$ otherwise. Prove the following identities:

   (a) $\chi_{A\cup B} = \chi_A + \chi_B - \chi_{A\cap B}$.

   (b) $\chi_{A\cap B} = \chi_A \chi_B$.

   This problem was briefly explained in class.

2. Let f and g be continuous on the region D. Deduce the inequality
$$2\iint_D |fg|\, dA \le \iint_D f^2\, dA + \iint_D g^2\, dA \ .$$
   Hint: Use $(a \pm b)^2 \ge 0$.

3. Let f be a non-negative continuous function on D and p a positive number. Show that
$$m \le \left(\frac{1}{|D|} \iint_D f^p\, dA \right)^{1/p} \le M \ ,$$
   where m and M are respectively the minimum and maximum of f and $|D|$ is the area of D.